Calibration of the SKA-low antenna array using drones

Loïc Van Hoorebeeck

March 25, 2019

UCL
Université catholique de Louvain

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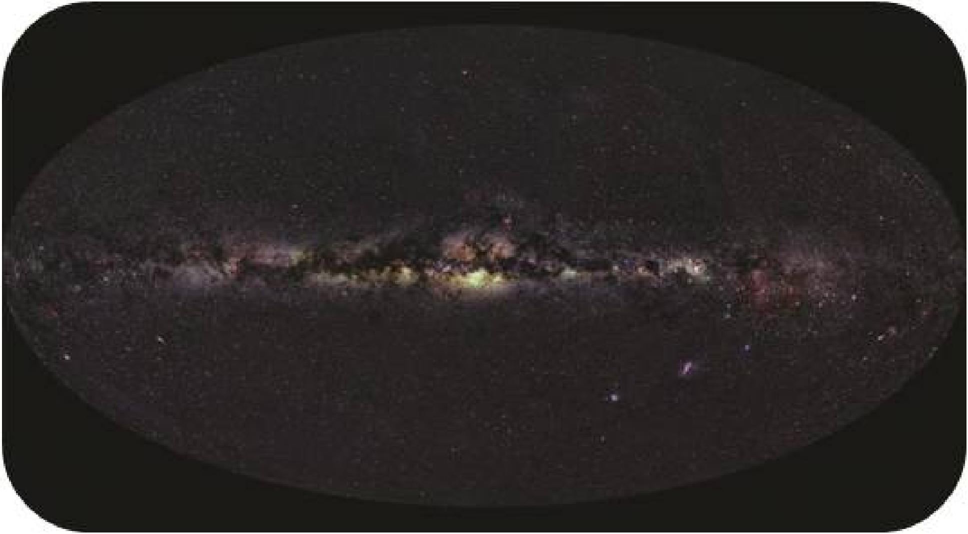
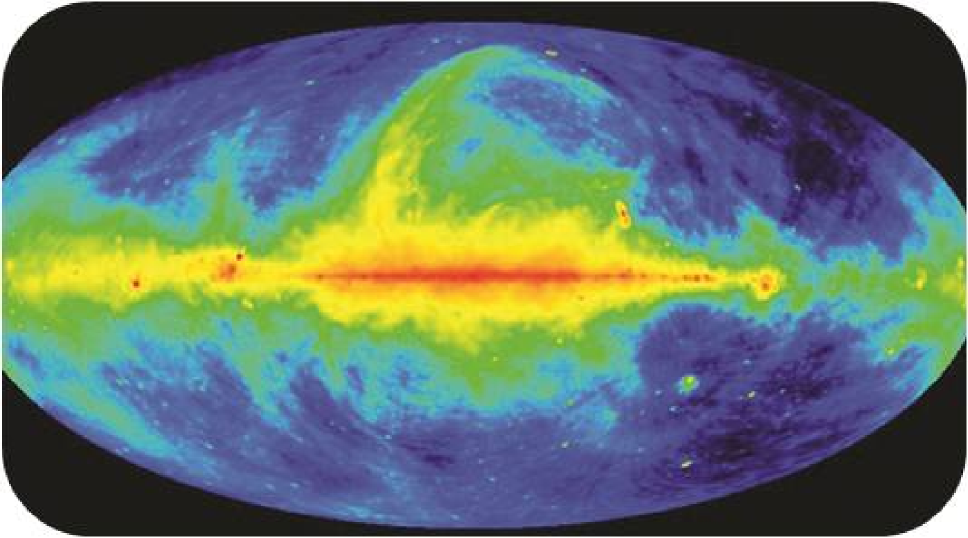
Optical telescope



Sources : Axel Mellinger/NASA SkyVie

Radio telescope



Sources : G. Haslam/MPIfR

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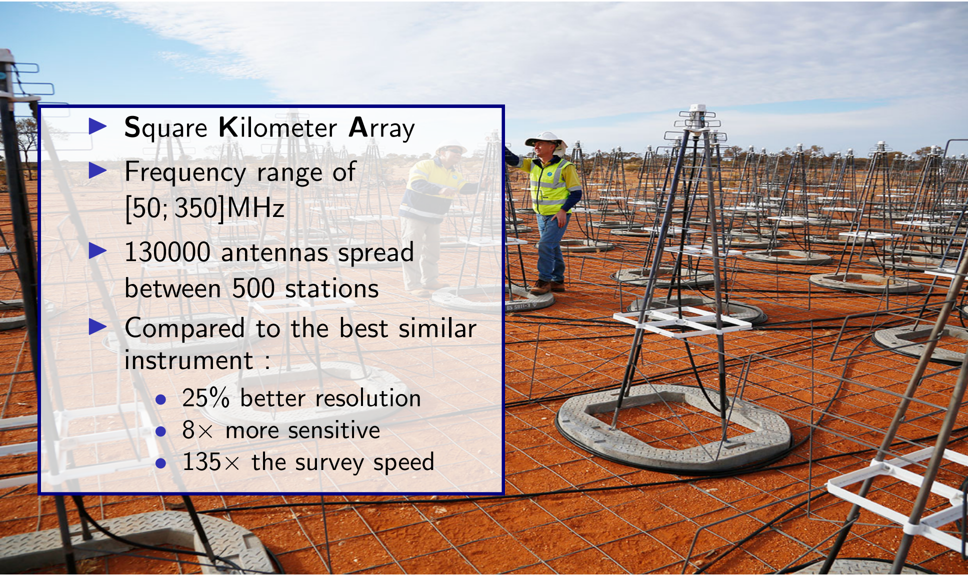

- **S**quare **K**ilometer **A**rray
- Frequency range of $[50; 350]$MHz
- 130000 antennas spread between 500 stations
- Compared to the best similar instrument :
  - 25% better resolution
  - $8\times$ more sensitive
  - $135\times$ the survey speed

Sources : https://spectrum.ieee.org https://www.skatelescope.org

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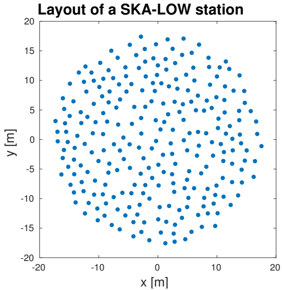
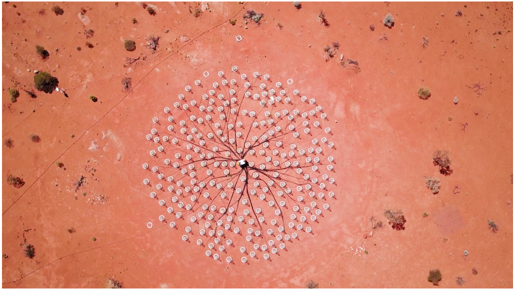
A first station overview





$N_a = 256$ antennas irregularly arranged. The Aperture Array Verification System 1 (AAVS1).

Sources : https://www.skatelescope.org

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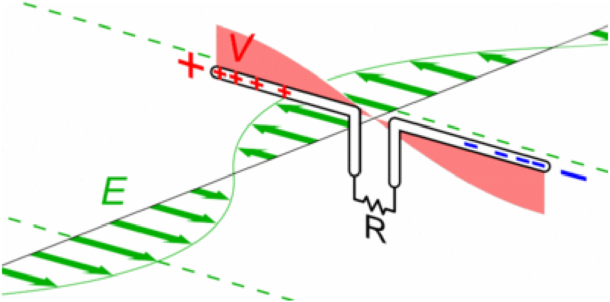
An example of a wired antenna



Sketch of the running of antenna by converting a sine electric current into a EM wave.

Sources : Wikipedia - Antenna

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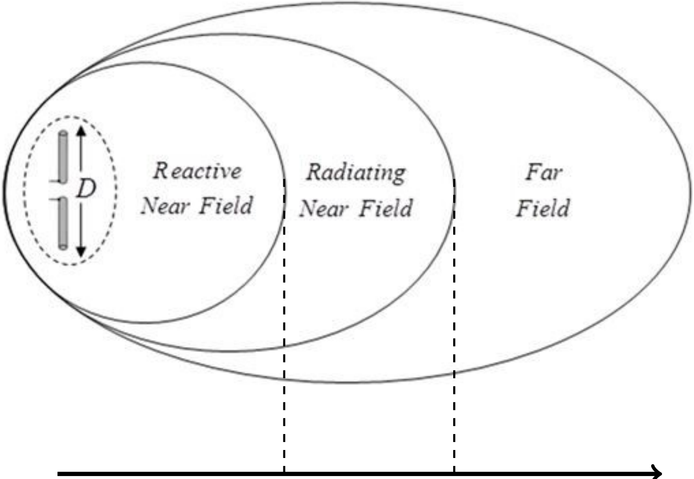
Field regions



$100\,\mathrm{m} \approx 0.62\sqrt{D^3/\lambda}$ $\qquad \underbrace{2D^2/\lambda}_{\text{Fraunhofer distance}} \approx 1\,\mathrm{km}$

Sources : Sefi Cohen Arazi, MTI Wireless Edge Ltd.

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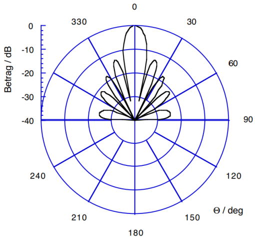
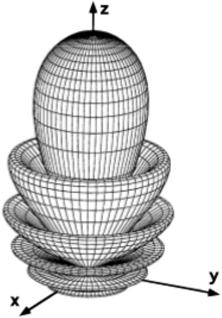
Radiation Pattern

$$\mathbf{F}(\theta,\varphi) = \lim_{\rho\to\infty} \frac{\mathbf{E}(\rho,\theta,\varphi)}{\max_{\theta,\varphi}\mathbf{E}(\rho,\theta,\varphi)} = F_v\,\mathbf{e}_\theta + F_h\,\mathbf{e}_\varphi$$

Embedded Element Pattern (EEP)

Radiation pattern when an antenna *i* is on with the other passively terminated.

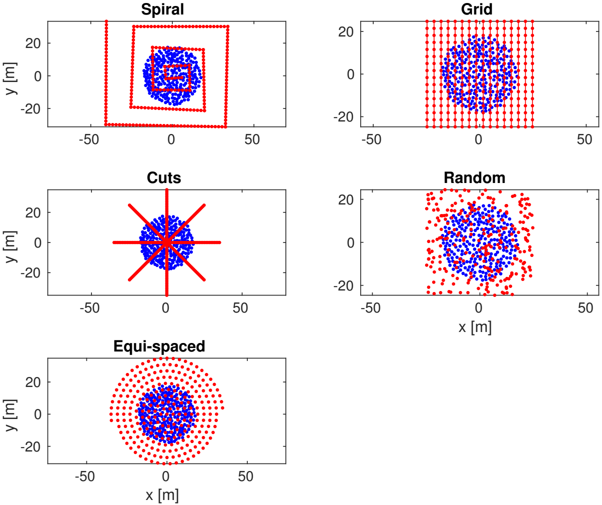




Sources : T. Zwick - Antennen und Mehrantennensysteme (KIT)

Calibration of the SKA-low antenna array using drones

Flight strategies



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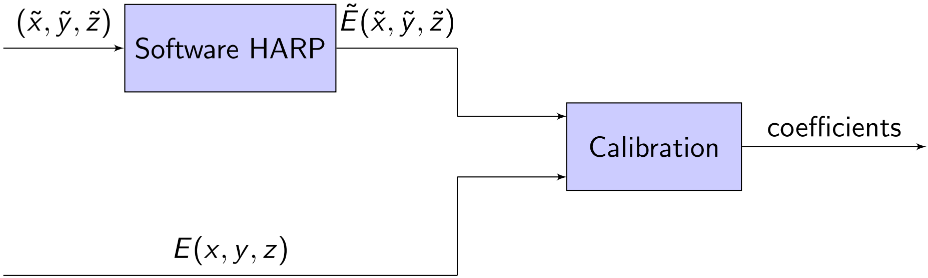
Calibration procedure



Calibration methods

Calibration of the SKA-low antenna array using drones

- Sum of every EEP of ON antenna i
- Computed pattern

$$\bar{\mathbf{F}}_v = \sum_{i=1}^{N_a} \underbrace{\sum_{j=1}^{N_a} \mathbf{B}^i_{v\,,\,j}\, c^i_j}_{\bar{\mathbf{F}}^i_v} \in \mathbb{C}^{N_e \times 1}$$

- Basis vectors $\in \mathbb{C}^{N_e \times N_a}$
- Calibration coefficients $\in \mathbb{C}^{N_a}$

Calibration of the SKA-low antenna array using drones

Formulation as N_a convex optimization problems

$$\min_{\mathbf{c}^i \in \mathbb{C}^{N_a}} ||\mathbf{F}_v^i - \bar{\mathbf{F}}_v^i||_2^2 + ||\mathbf{F}_h^i - \bar{\mathbf{F}}_h^i||_2^2 \qquad \forall i = 1 \ldots N_a$$

This is a **least-square** problem and its optimal solution satisfies

$$\begin{pmatrix} \mathbf{B}_h^i \\ \mathbf{B}_v^i \end{pmatrix}^{\mathrm{H}} \underbrace{\begin{pmatrix} \mathbf{B}_h^i \\ \mathbf{B}_v^i \end{pmatrix}}_{\mathbb{C}^{2\,N_{\mathrm{mes}} \times N_a}} \underbrace{\mathbf{c}^i}_{\mathbb{C}^{N_a}} = \begin{pmatrix} \mathbf{B}_h^i \\ \mathbf{B}_v^i \end{pmatrix}^{\mathrm{H}} \underbrace{\begin{pmatrix} \mathbf{F}_h^i \\ \mathbf{F}_v^i \end{pmatrix}}_{\mathbb{C}^{N_a}} \qquad \forall i = 1 \ldots N_a$$

Calibration of the SKA-low antenna array using drones

Error definition in dB

$$e_{\theta,\varphi} = 10\log_{10}\left(\left|\mathbf{F}_v - \bar{\mathbf{F}}_v\right|^2 + \left|\mathbf{F}_h - \bar{\mathbf{F}}_h\right|^2\right) - 10\log_{10}\max_{\theta,\varphi}\left\{|\mathbf{F}_v|^2 + |\mathbf{F}_h|^2\right\}$$

Metrics

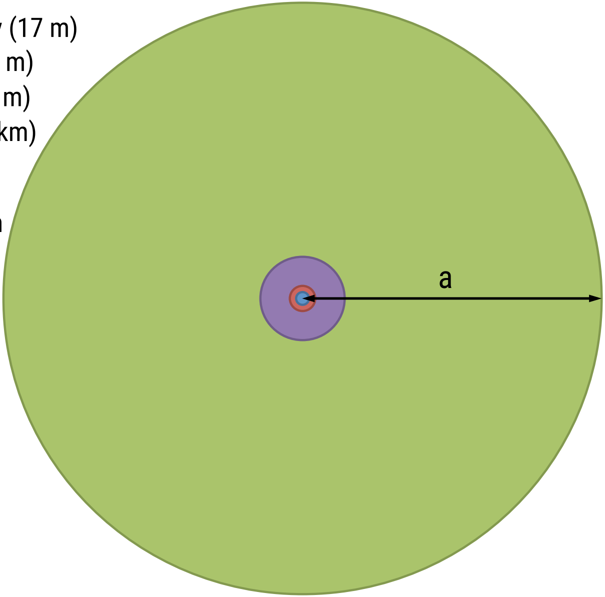
Maximum error	$e_M \triangleq \max_{\theta,\varphi} e_{\theta,\varphi}$
Mean error	$\mu_e \triangleq \mathbb{E}\{e_{\theta,\varphi}\}$

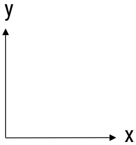
Results

Results

Antenna array (17 m)
Near field (25 m)
Far field (200 m)
Far field (1.7 km)

f = 140 MHz
a = distance from array center





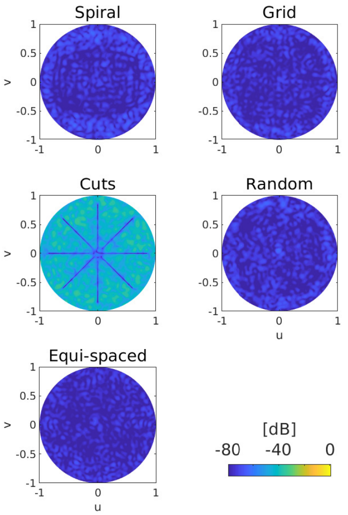
Far-field calibration

No flight restriction

- $h \triangleq 1\,\text{km}$ height to reach Fraunhofer distance
- $a = 1.7\,\text{km}$

Flight strategy	e_M (dB)	μ_e (dB)
Spiral	-60.9	-74
Grid	-63.4	-75.5
Cuts	-29.8	-45.3
Random	-60	-74.3
Equi-spaced	-64.3	-76.5

Good behavior for all strategies except cuts

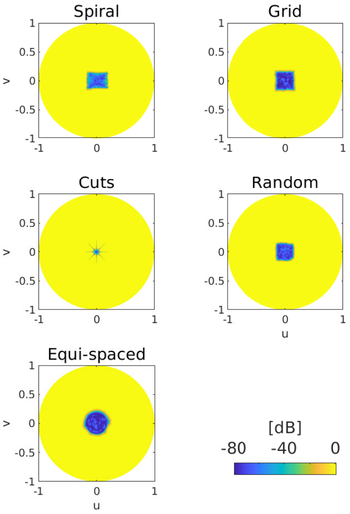


Far-field calibration

200 m flight restriction

- $h = 1\,\text{km}$
- $a = 200\,\text{m}$

Small error along the drone path
Large error anywhere else

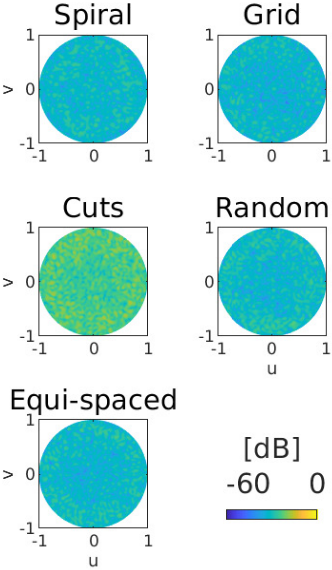


Near-field calibration

- $h = 10\,\mathrm{m}$
- $a = 25\,\mathrm{m}$

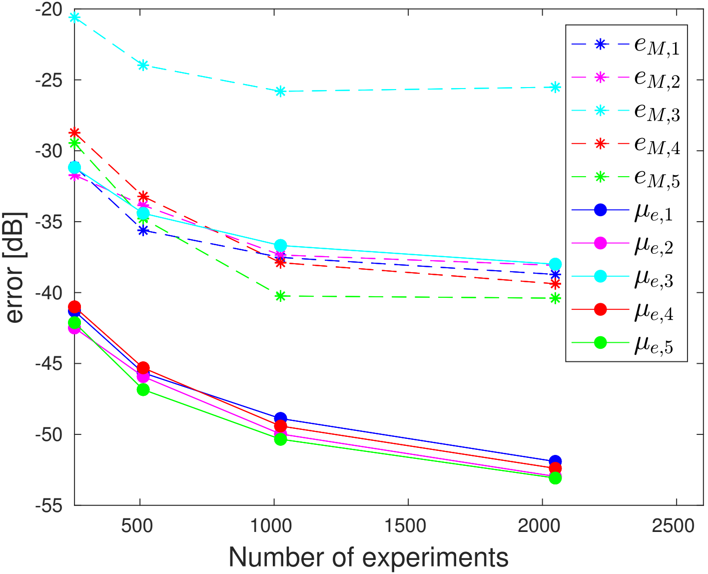
Flight strategy	e_M (dB)	μ_e (dB)
Spiral	-31.1	-41.3
Grid	-31.72	-42.5
Cuts	-20.6	-31.2
Random	-28.7	-41.0
Equi-spaced	-29.7	-41.1

Better than restricted FF but worse than unrestricted FF



Near Field

Impact of the number of experiments

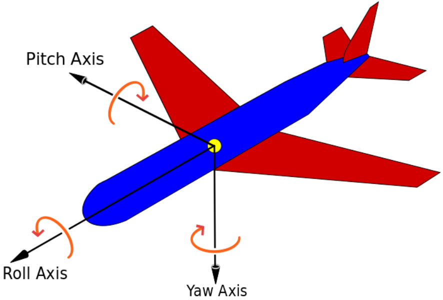


Near field

Drone attitude

Max error (dB)	Mean error (dB)	σ_y	σ_p	σ_r	(degrees)
-**48**.14	-57.38	**2**	2	2	
-**48**.46	-59.62	**2**	0	0	
-54.32	-65.46	0	2	0	
-54.32	-65.14	0	0	2	

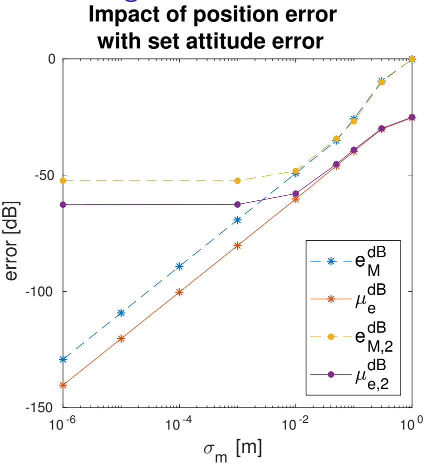
Smaller impact than position noise
Error mainly due to yaw angle

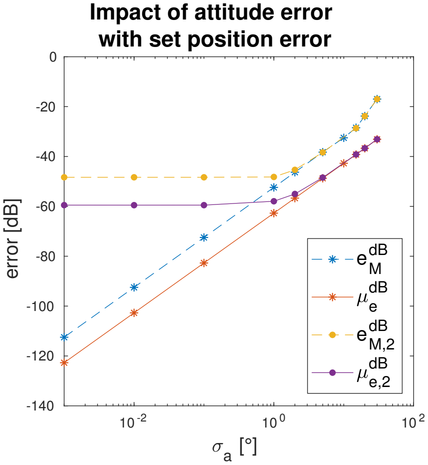


Source: Wikipedia

Near field

Combining both noises





Linear relationship between error and noise-level in dB scale
When combined, the error is dominated by the more significant noise

Regularization

Regularization

Mathematical formulation

$$\min_{\mathbf{x}\in\mathbb{R}^n} ||A\mathbf{x} - (\mathbf{b} + \epsilon\mathbf{f})||_2^2$$

with $\mathbf{A} \in \mathbb{R}^{m\times n}$, $\mathbf{b} \in \mathbb{R}^n$ the data matrix and $\epsilon\mathbf{f}$ the perturbation vector.

A numerical example

Let

$$A = \begin{pmatrix} 0.16 & 0.10 \\ 0.17 & 0.11 \\ 2.02 & 1.29 \end{pmatrix}, \quad \mathbf{x}^\star = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = A\mathbf{x}^\star$$

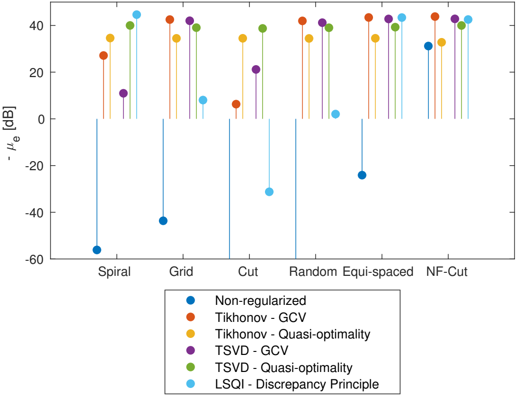
with $\mathbf{f} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ a perturbation with $\epsilon = 0.01$.

Moore-Penrose solution is $\mathbf{x}_\epsilon = \begin{pmatrix} 7.0 & -8.3 \end{pmatrix}^T$.

Regularization

Regularization type	Parameter choice strategy
Tikhonov	Discrepancy principle
Truncated SVD (TSVD)	Generalized cross-validation (GCD)
Least square minimization with a quadratic inequality constraint (LSQI)	Quasi-optimality criterion (QO)

Application on SKA-low calibration



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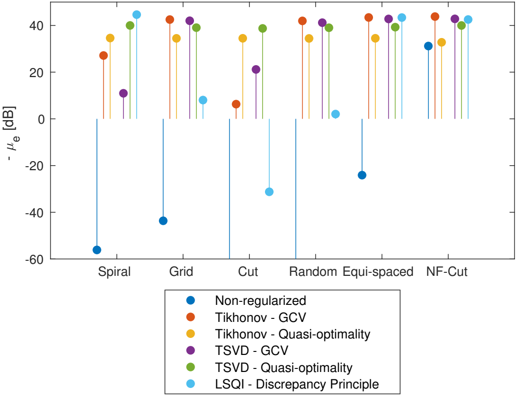

Conclusion and future works

Summary

Mean Error μ_e with 256 experiments

			Non regularized	Regularized				
				Tikh-GCV	Tikh-QO	TSVD-GCV	TSVD-QO	LSQI-DP
FF	Restricted	Spiral	56	-27	-35	-11	-40	-45
		Grid	44	-43	-34	-42	-39	-8
		Cut	88	-6	-34	-21	-39	31
		Random	63	-42	-34	-41	-39	-2
		Equi-spaced	24	-43	-35	-43	-39	-43
	Unrestricted	Spiral	-74					
		Grid	-75					
		Cut	-45					
		Random	-74					
		Equi-spaced	-76					
NF		Spiral	-41					
		Grid	-43					
		Cut	-31	-44	-33	-43	-40	-43
		Random	-41					
		Equi-spaced	-41					

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NF		Spiral	-41	-44	-33	-43	-40	-43
		Grid	-43					
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		Cut	**-31**	-44	-33	-43	**-40**	-43
		Random	-41					
		Equi-spaced	-41					

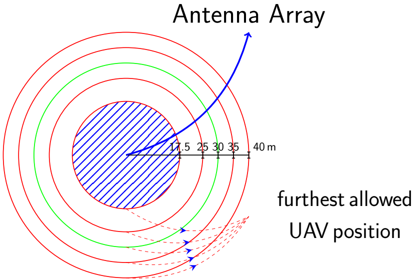
Going further

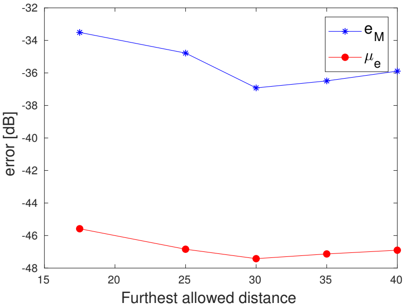
Extending the model

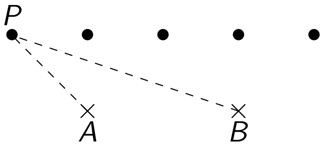
- Adding other noise sources (e.g. during the transmission)
- Taking finite ground plane effects into account

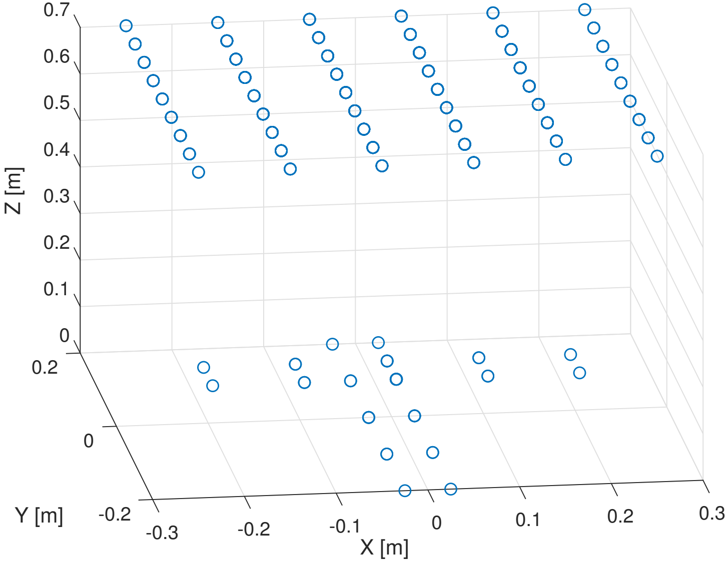
Generalization to $N_a \neq 256$ antennas

- FF case : $N_e \approx \max\{ \underbrace{50}_{\text{Sampling the pattern}} , \overbrace{N_a}^{\text{Unique solution}} \}$
- NF case : Every experiment yields N_a "independent" measurements
  $\Rightarrow$ Same behavior for all N_a


Antenna Array
17.5
25
30
35
40 m
furthest allowed
UAV position


-32
-34
-36
-38
-40
-42
-44
-46
-48
error [dB]
15
20
25
30
35
40
Furthest allowed distance
e_M
μ_e


P
A
B



Sources : Jean Cavillot UAV mesh.

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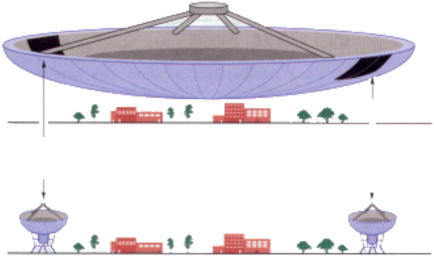
Why using an array of antennas ?

- Obtaining a better **angular resolution**
- Increasing the **sensitivity**

Interferometry : Emulation of a single larger dish



Sources : Introductory Astronomy - CCAC.