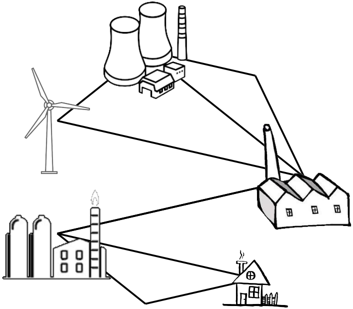
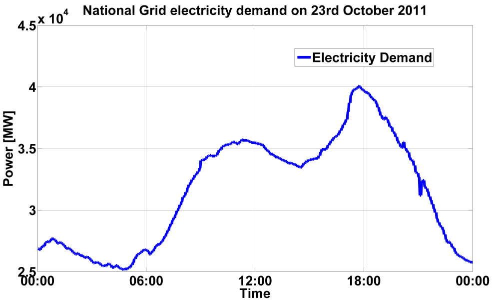
MILP-Based Algorithm for the Global Solution of Dynamic Economic Dispatch with Valve-Point Effects

Loïc Van Hoorebeeck, Anthony Papavasiliou, P.-A. Absil

August 28, 2020

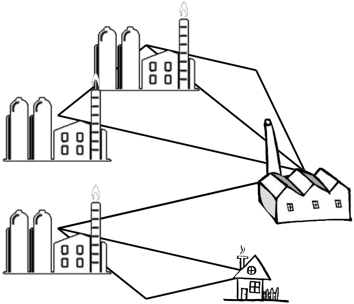
Economic Dispatch



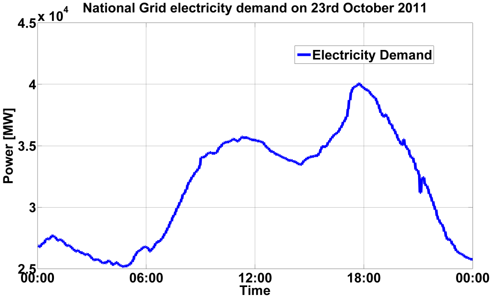


[The Grid 2025 Challenge – University of Glasgow]

Economic Dispatch



◎ Objective Taking into account the **valve point effect** which occurs in large multi-valves gas power plant.



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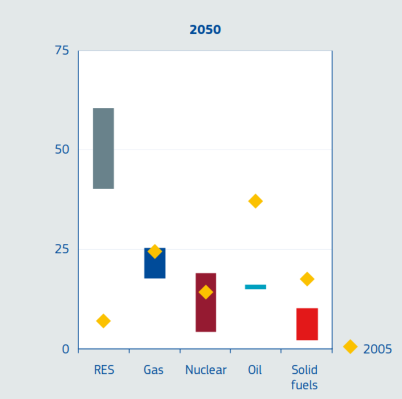
On the place of gas energy in **tomorrow's** power mix

European targets for 2030 and 2050



"Natural gas will continue to play a key role in the EU's energy mix in the coming years and gas can gain importance as the *back-up fuel* for variable electricity generation." (European Commission's Communication Energy 2020)

Source: Energy roadmap 2050

1. **Introduction**.

2. **Problem statement** - Economic Dispatch with Valve Point Effect.

3. **Description of the algorithm** - An Adaptive Piecewise-Linear Approximation.

4. **Study case** - A 10-units dispatch over 24 hours.

5. **Extension and further work.**

2. Problem statement

Data

Load demand: D_t $t\ 2\ T$

Spinning reserve: S_t $t\ 2\ T$

Set of producers with cost function f_i $i\ 2\ I$

Decision variables

Production: p_{it} $i\ 2\ I\ ,t\ 2\ T$

Reserve: s_{it} $i\ 2\ I\ ,t\ 2\ T$

Problem

How to optimally *dispatch* the power between producers ?

Optimization model

$$\min_{p_{it},\, s_{it}} \sum_{i=1,\, t=1}^{n,\, T} f_i(p_{it})$$ Fuel cost minimization

subject to

$$\sum_{i=1}^{n} p_{it} = D_t \ ,$$

$$\sum_{i=1}^{n} s_{it} \quad S_t \ ,$$

$$s_{it} \quad R_i^U \ ,$$

$$p_{it} + s_{it} \quad P_i^{\max} \ ,$$

$$P_i^{\min} \quad p_{it} \ ,$$

$$R_i^D \quad p_{it} \quad p_{i(t \quad 1)} \quad R_i^U \ .$$

Optimization model

$$\min_{p_{it},\, s_{it}} \sum_{i=1,\, t=1}^{n,\, T} f_i(p_{it})$$

subject to

$$\sum_{i=1}^{n} p_{it} = D_t \,,$$

Demand is met

$$\sum_{i=1}^{n} s_{it} \quad S_t \,,$$

Enough (up) spinning reserve

$$s_{it} \quad R_i^U \,,$$

$$p_{it} + s_{it} \quad P_i^{\max} \,,$$

$$P_i^{\min} \quad p_{it} \,,$$

$$R_i^D \quad p_{it} \quad p_{i(t\ 1)} \quad R_i^U \,.$$

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$$\sum_{i=1}^{n} s_{it} \quad S_t \,,$$

$$s_{it} \quad R_i^U \,,$$

Reserve cannot exceed the ramp constraint

$$p_{it} + s_{it} \quad P_i^{\max} \,,$$

$$P_i^{\min} \quad p_{it} \,,$$

$$R_i^D \quad p_{it} \quad p_{i(t \quad 1)} \quad R_i^U \,.$$

Optimization model

$$\min_{p_{it},\, s_{it}} \sum_{i=1,\, t=1}^{n,\, T} f_i(p_{it})$$

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$$s_{it} \quad R_i^U \,,$$

$$p_{it} + s_{it} \quad P_i^{\max} \,,$$

$$P_i^{\min} \quad p_{it} \,,$$

Restricted power range

$$R_i^D \quad p_{it} \quad p_{i(t \quad 1)} \quad R_i^U \,.$$

Optimization model

$$\min_{p_{it},\, s_{it}} \sum_{i=1,\, t=1}^{n,\, T} f_i(p_{it})$$

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Ramp constraints

Optimization model

$$\min_{p_{it},\, s_{it}} \sum_{i=1,\, t=1}^{n,\, T} f_i(p_{it})$$

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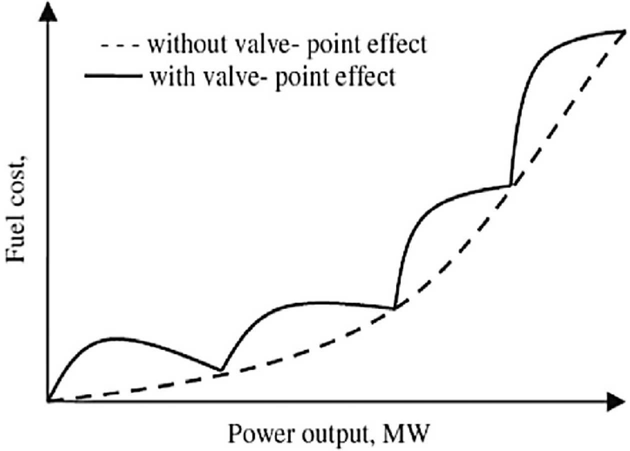
$$R_i^D \quad p_{it} \quad p_{i(t \quad 1)} \quad R_i^U \,.$$

Valve-Point Effect

The VPE is a natural characteristic of a gas turbine. Operating off a valve point increases the throttling losses, and therefore rises the heat rate.

$$f_i(p_{it}) = a_i\, p_{it}^2 + b_i\, p_{it} + c_i + d_i \left|\sin e_i(p_{it} \quad P_i^{\min})\right|$$



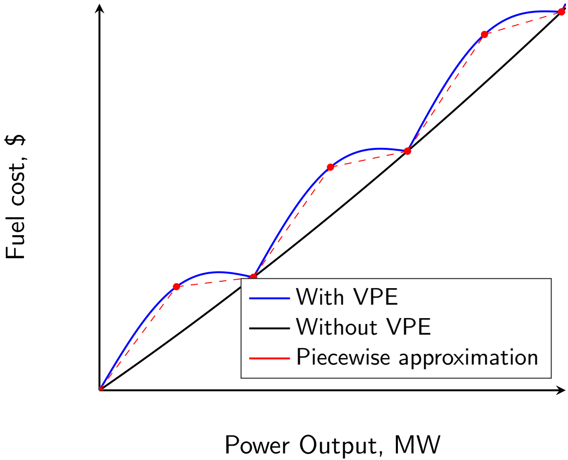
3. Description of the algorithm

Adaptive **P**iecewise-**L**inear **U**nder-**A**pproximation

Idea: a sequence of piecewise approximations.

We could use a uniform grid...

... but there are too many integer variables!



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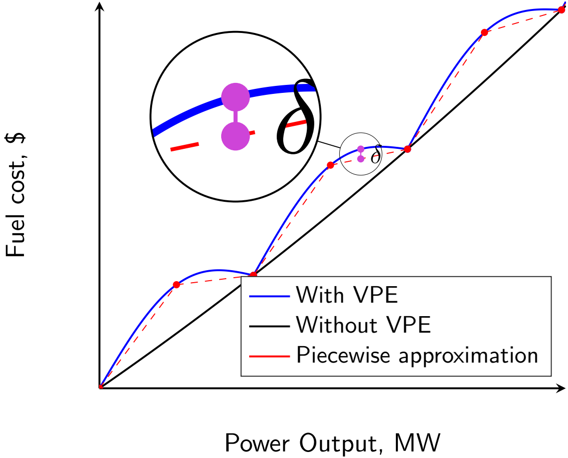

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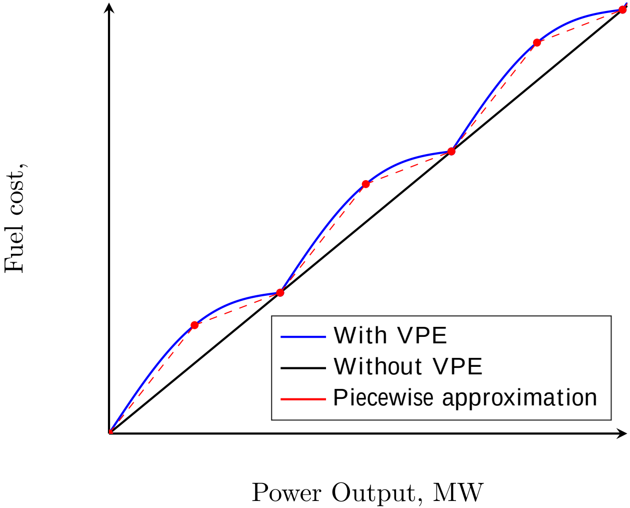

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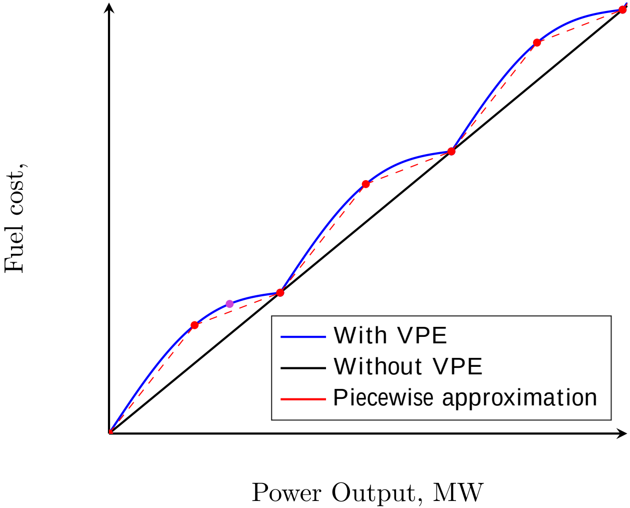

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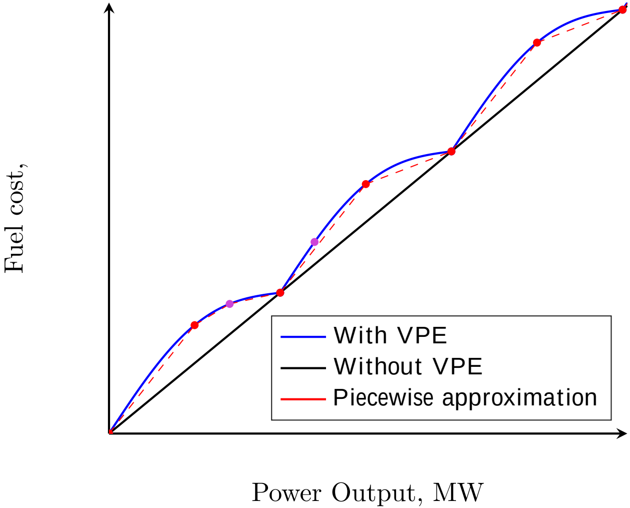

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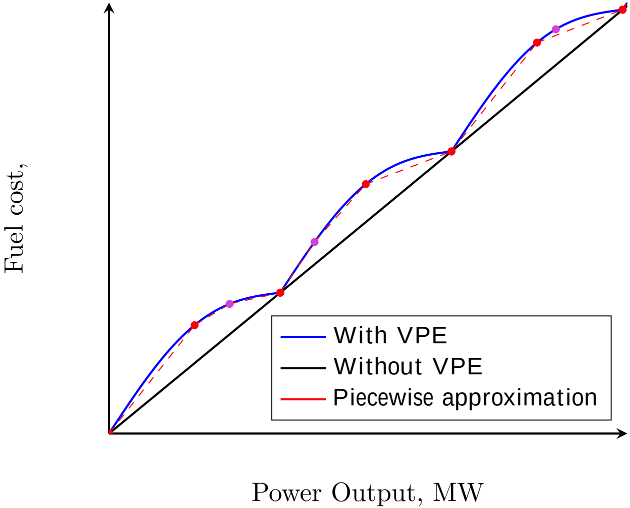

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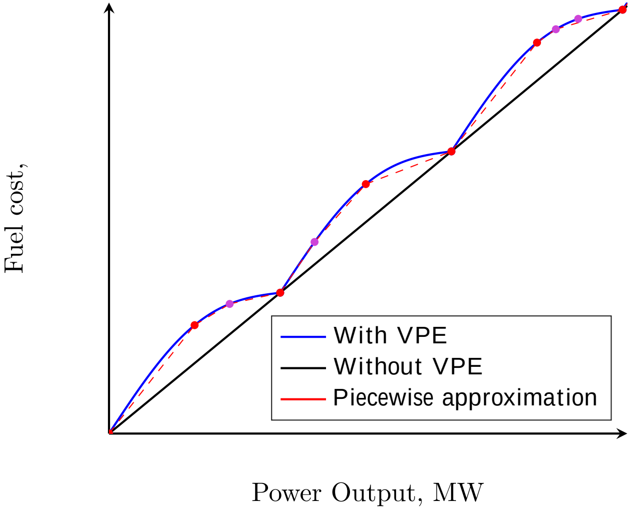

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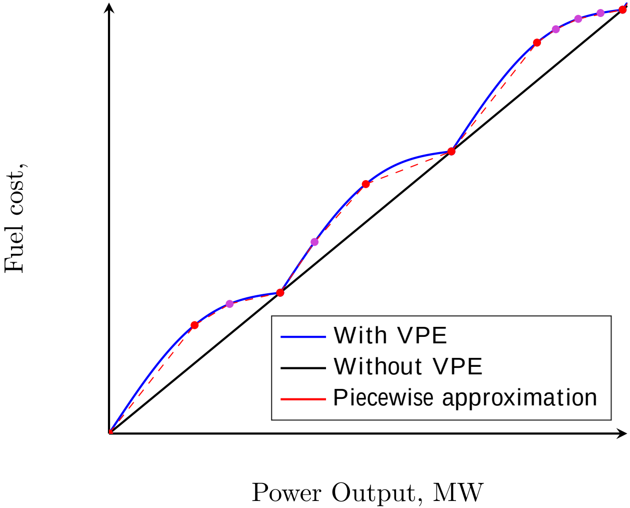
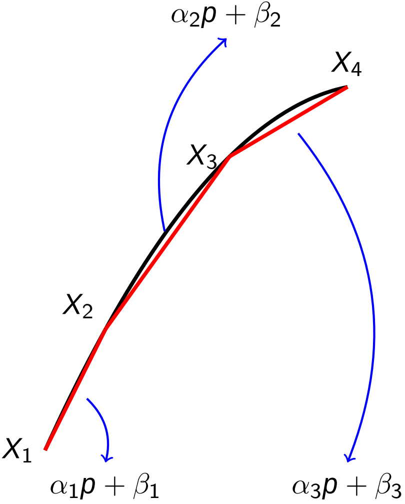


Piecewise linearization of objective

First model: binary variables

$$g(p, \boldsymbol{\xi}, \boldsymbol{\eta}) := \sum_j \alpha_j \, \xi_j + \beta_j \, \eta_j$$

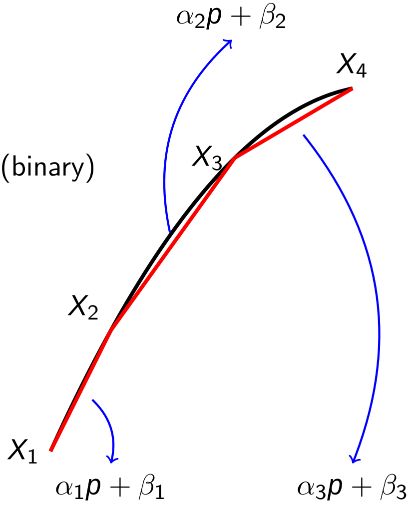


Piecewise linearization of objective

First model: binary variables

$\sum_j \eta_j = 1$ (binary)

$$g(p, \boldsymbol{\xi}, \boldsymbol{\eta}) := \sum_j \alpha_j \, \xi_j + \beta_j \, \eta_j$$

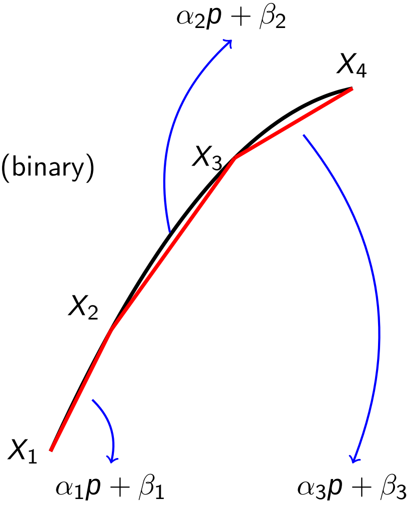


Piecewise linearization of objective

First model: binary variables

$\sum_j \xi_j = p$ (continuous) $\sum_j \eta_j = 1$ (binary)

$$g(p, \boldsymbol{\xi}, \boldsymbol{\eta}) := \sum_j \alpha_j \xi_j + \beta_j \eta_j$$



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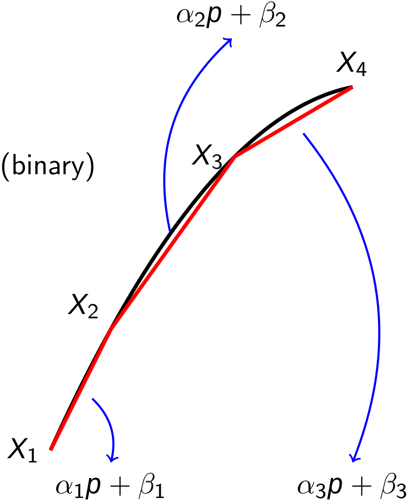
First model: binary variables

$\sum_j \xi_j = p$ (continuous) $\sum_j \eta_j = 1$ (binary)

$$g(p, \boldsymbol{\xi}, \boldsymbol{\eta}) := \sum_j \alpha_j \, \xi_j + \beta_j \, \eta_j$$

and s.t. $X_j \, \eta_j \quad \xi_j \quad X_{j+1} \, \eta_j$

Exactly one η_j and associated ξ_j selected.



Optimality gap

- Gap between surrogate and true solution

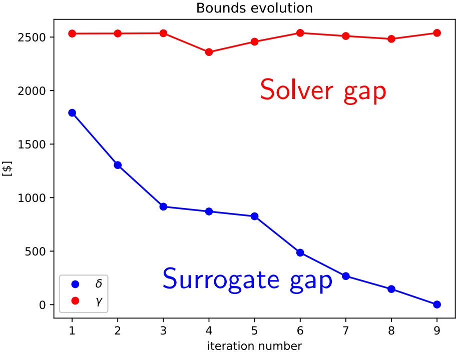
$$\overbrace{\left| f\,(\mathbf{p}\ \ ) \quad \mathbf{f}(\mathbf{p}^{\mathbf{k}}) \right|}^{\text{Optimality gap}} \qquad \delta^{k} + \gamma^{k} + \epsilon^{k}$$

- Solver tolerance error
- Over-approximation error

What about the convergence?

- γ^k is bounded below by $\gamma f(\mathbf{p}\ )$;
- ϵ^k is virtually negligible since the "convex zones" are smaller than 0.1% of the domain ;
- δ^k converges to zero.

A practical example



Van Hoorebeeck *et. al.*, 2019. (Preprint)

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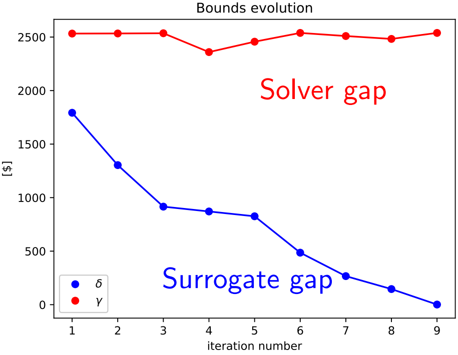


In general

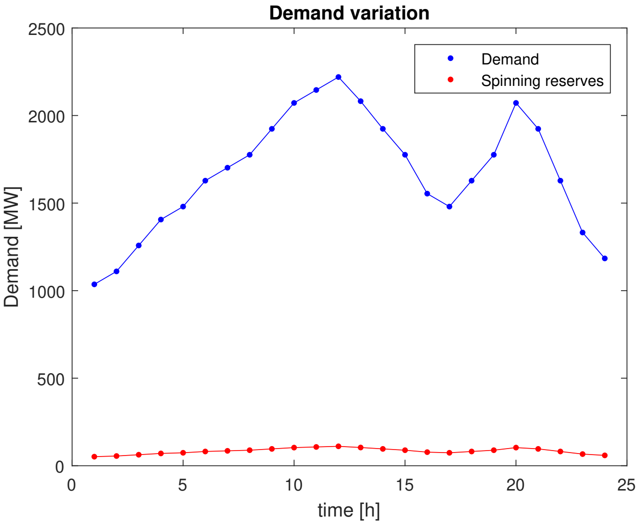
Theorem 1

For L-continuous piecewise-concave cost functions,

$$\lim_{k \to \infty} \delta^k = 0 .$$

Van Hoorebeeck *et. al.*, 2019. (Preprint)

4. Study case - A 10-units dispatch over 24 hours



Spinning reserves set at 5% of the demand.
10 units with valve-point loading effect.

Results table

Previous results

Method	Total generation cost ($)			S-time(min)
	Minimum	Average	Maximum	
SQP [3]	1051163	NA	NA	0.42
EP [3]	1048638	NA	NA	15.05
CDE [16]	1036756	1040586	1452558	0.20
TVAC-IPSO[19]	1018217	1018965	1020418	2.72
HBPSO[31]	1018159	1019850	1021813	3.09
CSO [25]	1017660	1018120	1019286	0.90
EBSO [21]	1017147	1017526	1017891	0.15
MILP	1016316			0.94
MILP-IPM	1016311			1.02

And our approach ?

Pan *et. al.*, 2018.

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And our approach ?

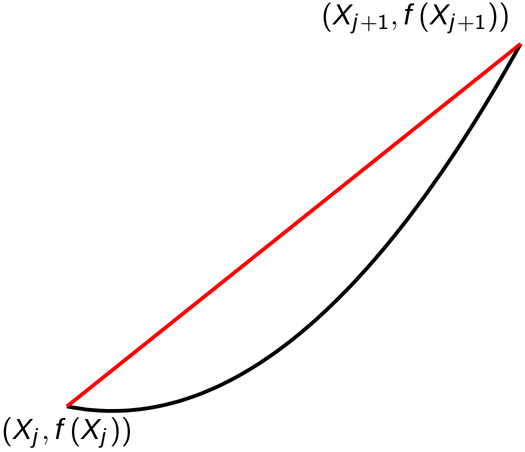
APLUA	1016276$	(1013410)	15(min)
APLUA + Local Heuristic	1016207$	(1014719)	1.5(min)

Pan *et. al.*, 2018.

5. Extension and further work

Important characteristic of the method: **Under**-approximation

) The method is not valid for convex functions (e.g. without valve point effect)



5. Extension and further work

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Assume f_1 convex and f_2 piecewise concave

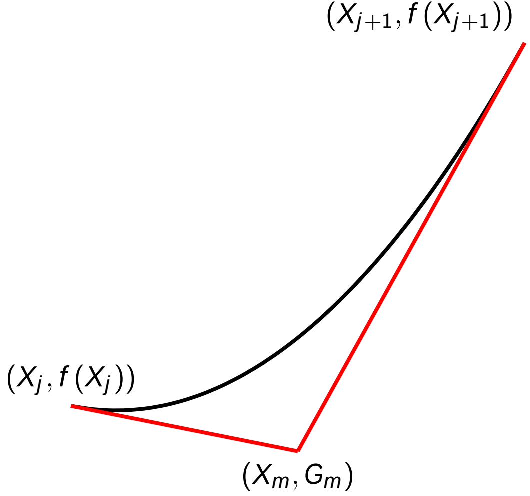
- Feed the solver with the full convex functions;(L)
- Under-approximate the convex functions.(R)

$$\min_{x \in C} f_1 + f_2$$

(L) (R)

$$\min_{x \in C} f_1 + g_2 \qquad \min_{x \in C} g_1 + g_2$$

Under-approximation of a convex function



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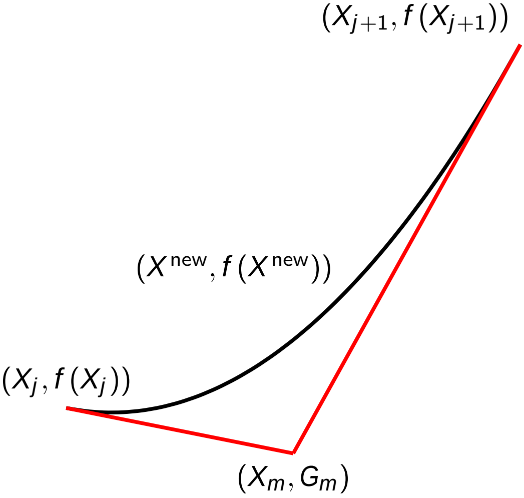

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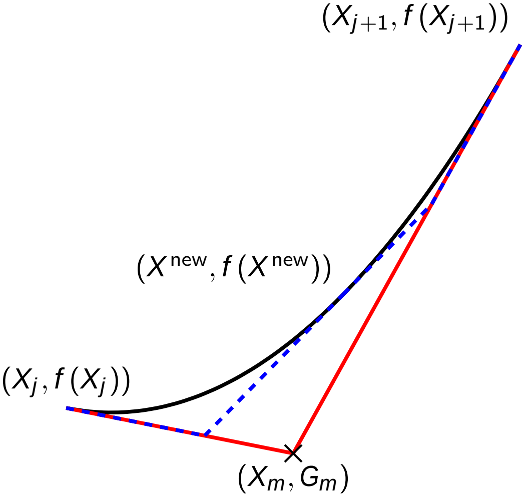

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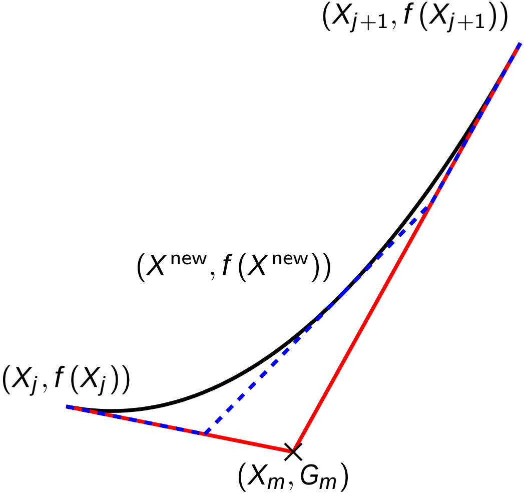


- Possible to prove that g^{k+1} g^k and that we cannot do better with that number of points
- Number of integer variables rises linearly (factor 2)

Power losses and network constraints

(Revisited) demand constraints

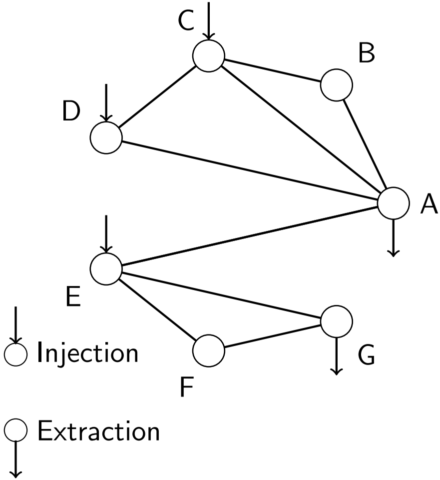
$$\sum_{i=1}^{n} p_{it} = D_t + p^L(\mathbf{p}_t)$$

$p^L(\mathbf{p}_t)$ models the transmission losses computed as

$$p^L(\mathbf{p}_t) = \mathbf{p}_t^T \mathbf{B} \mathbf{p}_t + \mathbf{B}_0 \mathbf{p}_t + \mathbf{B}_{00}$$

with **B** symmetric matrix.

Network constraints



Conclusion

- APLUA manages to find a good candidate ...
- ... but it takes more time ...
- ... and we are limited by the solver tolerance gap ...
- ... however we provide a lower bound.
- ? How to take the quadratic transmission losses into account?

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