Global Solution of Economic Dispatch with Valve Point Effects and Transmission Constraints

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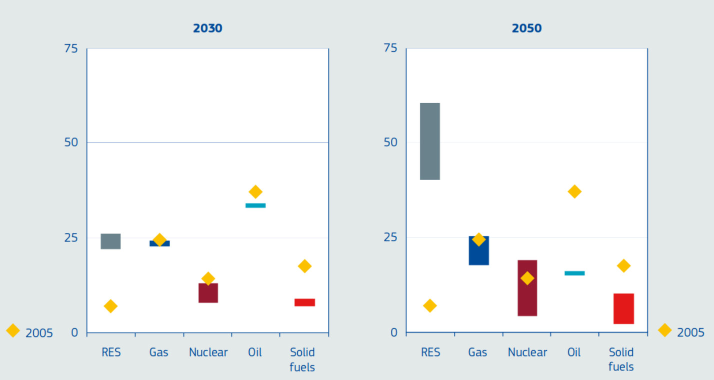
PSCC 2020

On the place of gas energy in tomorrow's power mix

European targets for 2030 and 2050



"Natural gas will continue to play a key role in the EU's energy mix in the coming years and gas can gain importance as the *back-up fuel* for variable electricity generation." (European Commission's Communication Energy 2020)

Source: European Energy roadmap 2050

1. **Problem statement** - Economic Dispatch with Valve Point Effect.

2. **Methods** - Adaptive piecewise linearization.
   - Exact method
   - Heuristic

3. **Convergence Analysis** - Optimality gap and other bounds.

4. **Study Cases** - modified IEEE-57 and IEEE-118 bus case.

5. **Conclusion**

Problem Statement

2. Problem statement - Economic Dispatch with Valve Point Effect.

Data

Set of bus nodes: N

Set of lines: K

Load demand: D_{nt} $\quad n \in N, t \in T$

Set of producers with cost function f_g $\quad g \in G$

Ramp constraint: R_g^+, R_g^- $\quad g \in G$

Max flow constraint: TC_k $\quad k \in K$

Decision variables

Production: p_{gt} $\quad g \in G, t \in T$

Flow: e_{kt} $\quad k \in K, t \in T$

Problem

How to optimally *dispatch* the power between producers ?

Optimization model

$$\min_{\boldsymbol{p},\boldsymbol{e}} \sum_{g=1,\, t=1} f_g(p_{gt})$$

Fuel cost minimization

s.t.

$$p_{gt} \leq P_g^+ \,,$$

$$P_g^- \leq p_{gt} \,,$$

$$-\sum_{g \in G_n} p_{gt} - \sum_{k=(\cdot,n)} e_{kt} + D_{nt} + \sum_{k=(n,\cdot)} e_{kt} = 0 \,,$$

$$-R_g^- \leq p_{gt} - p_{g(t-1)} \leq R_g^+ \,,$$

$$-TC_k \leq e_{kt} \leq TC_k \,.$$

Nomenclature

Bus nodes: N

Cost: f_g

Flow: e_{kt}

Lines: K

Load: D_{nt}

Max flow: TC_k

Production: p_{gt}

Ramp: R_g^+, R_g^-

Optimization model

$$\min_{\boldsymbol{p},\,\boldsymbol{e}} \sum_{g=1,\,t=1} f_g(p_{gt})$$

s.t.

$$p_{gt} \leq P_g^+ \,,$$

$$P_g^- \leq p_{gt} \,,$$

Restricted power range

$$-\sum_{g \in G_n} p_{gt} - \sum_{k=(\cdot,n)} e_{kt} + D_{nt} + \sum_{k=(n,\cdot)} e_{kt} = 0 \,,$$

$$-R_g^- \leq p_{gt} - p_{g(t-1)} \leq R_g^+ \,,$$

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Flow conservation

$$-\sum_{g\in G_n} p_{gt} - \sum_{k=(\cdot,n)} e_{kt} + D_{nt} + \sum_{k=(n,\cdot)} e_{kt} = 0 \,,$$

$$-R_g^- \leq p_{gt} - p_{g(t-1)} \leq R_g^+ \,,$$

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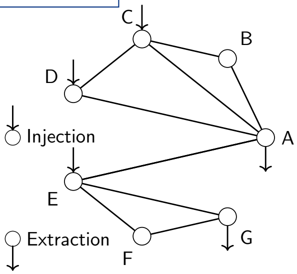

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Ramp constraints

$$-TC_k \leq e_{kt} \leq TC_k \,.$$

Nomenclature
Bus nodes: N
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Lines: K
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Flow can not exceed the limit

Nomenclature

Bus nodes: N

Cost: f_g

Flow: e_{kt}

Lines: K

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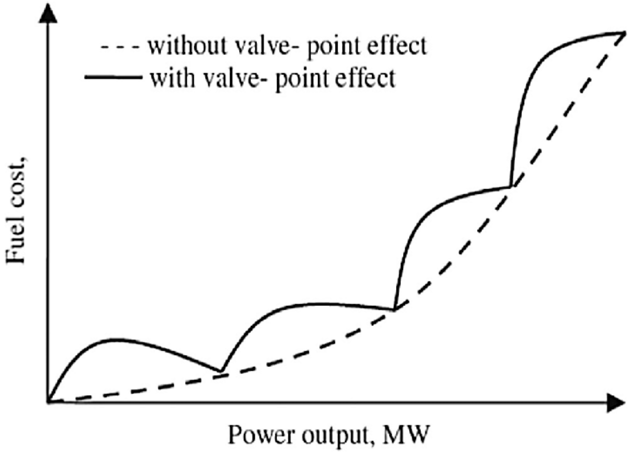
Valve-Point Effect

The VPE is a natural characteristic of a gas turbine. Operating off a valve point increases the throttling losses, therefore rising the heat rate.

$$f_g(p_{gt}) = \underbrace{A_g p_{gt}^2 + B_g p_{gt} + C_g}_{:=f_g^{\mathrm{Q}}(p_{gt})} + \underbrace{D_g \left|\sin E_g(p_{gt} - P_g^-)\right|}_{:=f_g^{\mathrm{VPE}}(p_{gt})} .$$



Methods

Exact Method

3. Methods - Adaptive piecewise linearization

Original problem

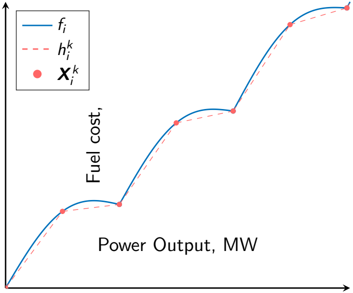
$$(P) \quad \min_{\boldsymbol{x}\in\Omega} f(\boldsymbol{x}) = \sum_i f_i(x_i)$$

- Feasible set is a polyhedron
- Objective function is non-convex and non-smooth
- Optimal solution $(\boldsymbol{x}^*, f(\boldsymbol{x}^*))$

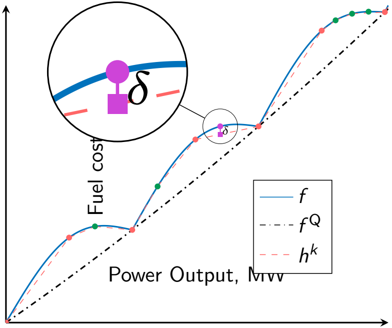
Surrogate problem

$$(S)^k \quad \min_{\boldsymbol{x}\in\Omega} h^k(\boldsymbol{x}) = \sum_i h_i^k(x_i)$$

- Feasible set is unchanged
- Objective function is piecewise linear defined with *knots* $\boldsymbol{X}^k$
- Optimal solution $(\boldsymbol{x}_k^{**}, h^k(\boldsymbol{x}_k^{**}))$



An algorithm for the global solution

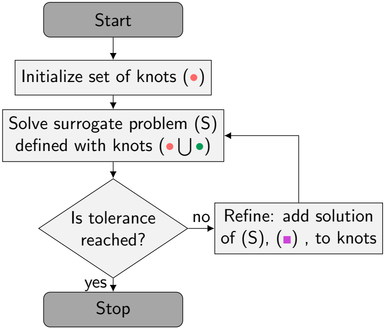


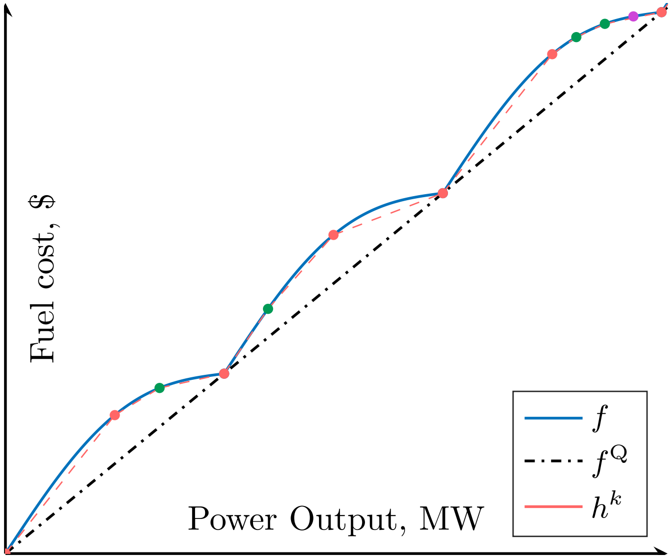
■ Optimal solution of $(S)^k$: $(x_k^{**}, h^k(x_k^{**}))$

● Initial set of knots: $\boldsymbol{X}_0$

● Previously added knots: $(x_l^{**}, f(x_l^{**}))$ for $l = 1 \ldots k-1$

● Knot to be added: $(x_k^{**}, f(x_k^{**}))$





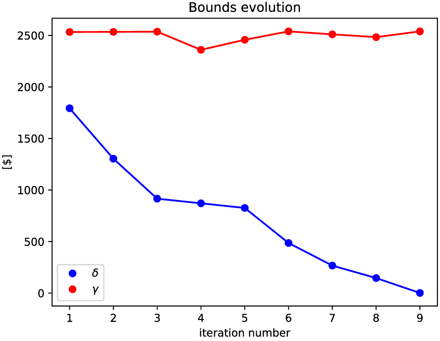
Fuel cost, $
Power Output, MW
f
f^{Q}
h^{k}

Bounds evolution

Theorem

For Lipshitz continuous cost function f, the sequence of iterations provided by APLA satisfies

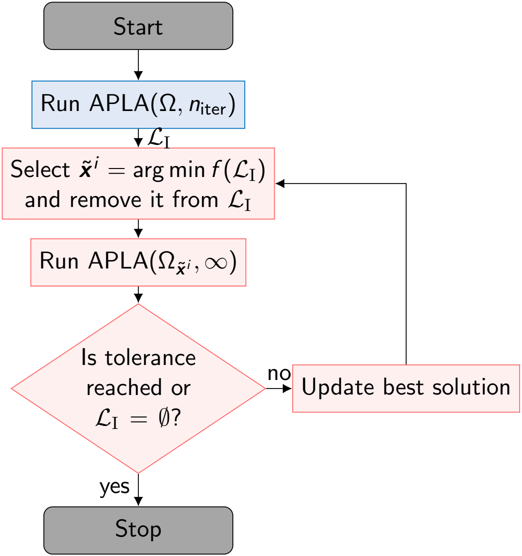
$$\lim_{k \to \infty} \delta^k = 0\,.$$



Methods

Heuristic

Heuristic flowchart



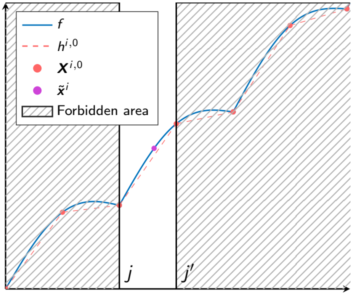
- Optimization run on the whole domain Ω to improve γ^k, $k \in 1 \dots n_{\text{iter}}$.
- Local improvement of each candidate $\tilde{\boldsymbol{x}}^i \in \mathcal{L}_\mathrm{I}$ obtained in previous step. Only δ can be improved.

Domain restriction

Let $\tilde{\boldsymbol{x}}^i = \arg\min f(\mathcal{L}_\mathrm{I})$, $\Omega_{\tilde{\boldsymbol{x}}^i}$ is the local restriction of the domain around $\tilde{\boldsymbol{x}}^i$. The power generation range becomes

$$X^i_{gtj} \leq p_{gt} \leq X^i_{gtj'}$$

for $j < j'$.

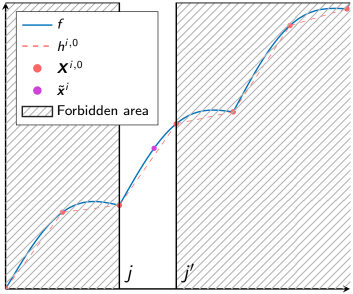


Domain restriction

The surrogate problem $(S)^{i,\tilde{k}}$ becomes

$$\begin{array}{ll} (S)^{i,\tilde{k}} & \min h^{i,\tilde{k}}(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in \Omega_{\tilde{\boldsymbol{x}}^i} \end{array}$$

- Easier as the number of integer variables decreases drastically ($j = j' - 1 \Rightarrow (S)^{i,0}$ is a (LP))
- Global optimal solution may lie outside $\Omega_{\tilde{\boldsymbol{x}}^i}$
- Any lower bound is not a valid lower bound for (P)



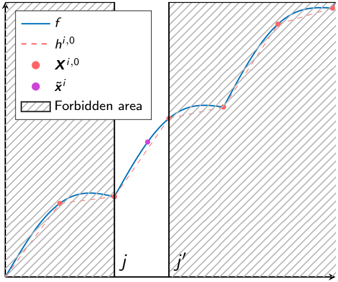
Domain restriction

Local heuristic - H-local

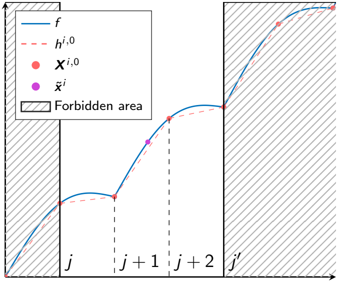
"Full" heuristic - H-full

$$(S)^{i,\tilde{k}} \quad \min_{\boldsymbol{x} \in \Omega_{\tilde{x}^i}} h^{i,\tilde{k}}(\boldsymbol{x})$$

- Single intervalle support
- $j' = j + 1.$



- Multi intervalles support
- $j' = j + 3.$



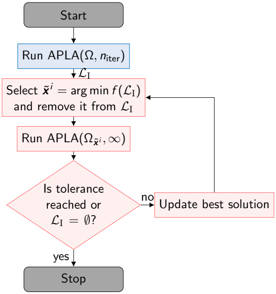
Convergence Analysis

Optimality gap of the heuristic solution (i)

- Objective value at iteration $\tilde{k}$ of subproblem i

$$f(\boldsymbol{x}^{i,\tilde{k}}) - f(\boldsymbol{x}^*) \leq \overbrace{f(\boldsymbol{x}^{i,\tilde{k}}) - h^{n_{\text{iter}}}(\boldsymbol{x}^{n_{\text{iter}}}) + \gamma^{n_{\text{iter}}}}^{\text{Optimality gap}}$$

- Surrogate value at iteration n_{iter} on whole domain
- Solver tolerance



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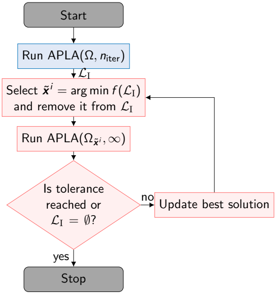
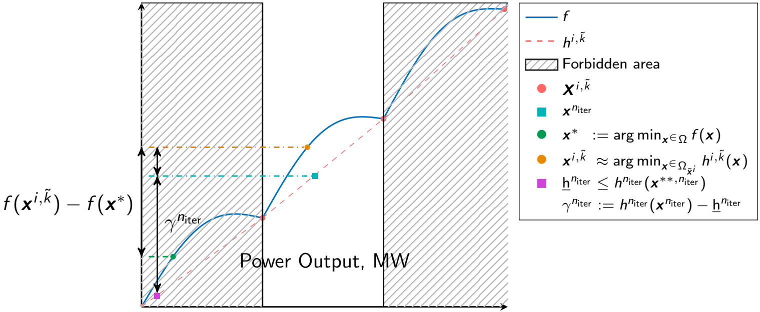


No convergence guarantee
Once we reach the inner loop, the lower bound remains unchanged.

A practical visualization (i)

$$f(\boldsymbol{x}^{i,\tilde{k}}) - f(\boldsymbol{x}^*) \leq f(\boldsymbol{x}^{i,\tilde{k}}) - h^{n_{\text{iter}}}(\boldsymbol{x}^{n_{\text{iter}}}) + \gamma^{n_{\text{iter}}} = f(\boldsymbol{x}^{i,\tilde{k}}) - \underline{\text{h}}^{n_{\text{iter}}}$$

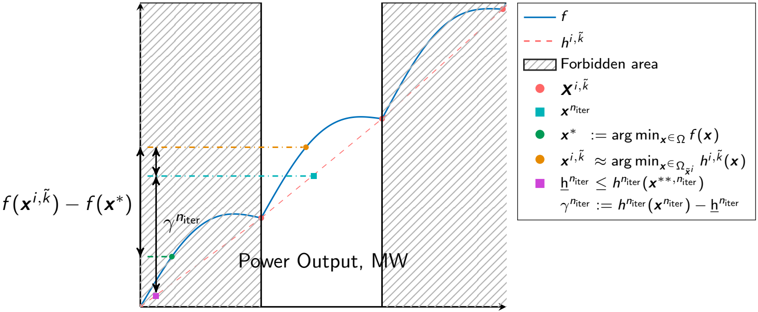


$$\underline{\text{h}}^{n_{\text{iter}}} \leq h^{n_{\text{iter}}}(\boldsymbol{x}^{**,n_{\text{iter}}}) \leq h^{n_{\text{iter}}}(\boldsymbol{x}^*) \leq f(\boldsymbol{x}^*)$$

$$\boxed{-f(\boldsymbol{x}^*) \leq -\underline{\text{h}}^{n_{\text{iter}}}}$$

A practical visualization (i)

$$f(\boldsymbol{x}^{i,\tilde{k}}) - f(\boldsymbol{x}^*) \leq f(\boldsymbol{x}^{i,\tilde{k}}) - h^{n_{\text{iter}}}(\boldsymbol{x}^{n_{\text{iter}}}) + \gamma^{n_{\text{iter}}} = f(\boldsymbol{x}^{i,\tilde{k}}) - \underline{\mathrm{h}}^{n_{\text{iter}}}$$



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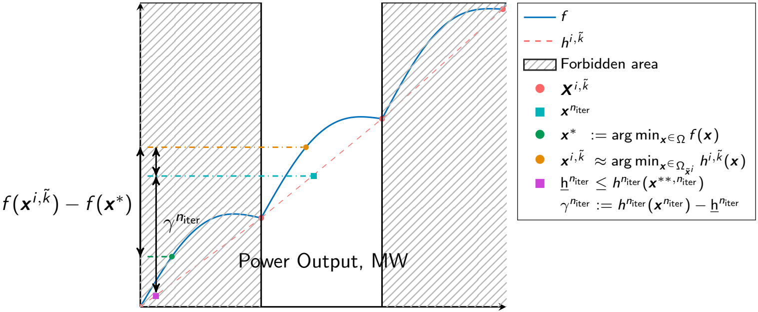


$$\underline{\text{h}}^{n_{\text{iter}}} \leq h^{n_{\text{iter}}}(\boldsymbol{x}^{**,n_{\text{iter}}}) \leq h^{n_{\text{iter}}}(\boldsymbol{x}^*) \leq f(\boldsymbol{x}^*)$$

$$\boxed{-f(\boldsymbol{x}^*) \leq -\underline{\text{h}}^{n_{\text{iter}}}}$$

Optimality gap of the heuristic solution (ii)

Could we obtain an expression in term of δ ?

$$
\begin{aligned}
f(\boldsymbol{x}^{i,\tilde{k}}) - f(\boldsymbol{x}^*) &= f(\boldsymbol{x}^{i,\tilde{k}}) - f(\boldsymbol{x}^{*,i}) + \underbrace{f(\boldsymbol{x}^{*,i}) - f(\boldsymbol{x}^*)}_{:=\zeta^i} \\
&\leq \delta^{i,\tilde{k}} + \gamma^{i,\tilde{k}} + \zeta^i \qquad (1) \\
&\approx \delta^{i,\tilde{k}} + \zeta^i \qquad (2)
\end{aligned}
$$

Nomenclature
$\boldsymbol{x}^* := \arg\min_{\boldsymbol{x}\in\Omega} f(\boldsymbol{x})$
$\boldsymbol{x}^{*,i} := \arg\min_{\boldsymbol{x}\in\Omega_{\tilde{\boldsymbol{x}}^i}} f(\boldsymbol{x})$

(1) APLA applied to the restricted domain $\Omega_{\tilde{\boldsymbol{x}}}$ is a valid instance.

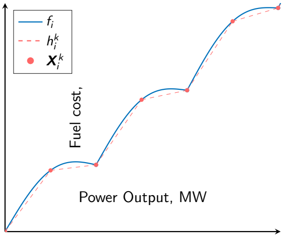
(2) $(S)^{i,\tilde{k}}$ solved to optimality.

Comparison between APLA and the heuristic

APLA

$$(S)^k \quad \min_{\boldsymbol{x}\in\Omega} h^k(\boldsymbol{x})$$

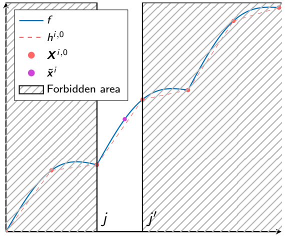
- $f(\boldsymbol{x}^k) - f(\boldsymbol{x}^*) \leq \delta^k + \gamma^k$
- $\lim_{k\to\infty} f(\boldsymbol{x}^k) - f(\boldsymbol{x}^*) \leq \gamma$
- Global optimal solution obtained up to the solver accuracy



Heuristic

$$(S)^{i,\tilde{k}} \quad \min_{\boldsymbol{x}\in\Omega_{\tilde{\boldsymbol{x}}^i}} h^{i,\tilde{k}}(\boldsymbol{x})$$

- $f(\boldsymbol{x}^{i,\tilde{k}}) - f(\boldsymbol{x}^*) \leq \delta^{i,\tilde{k}} + \zeta^i$
- $\lim_{\tilde{k}\to\infty} f(\boldsymbol{x}^{i,\tilde{k}}) - f(\boldsymbol{x}^*) \leq \zeta^i$
- Best solution bounded by $\zeta^i \geq 0$

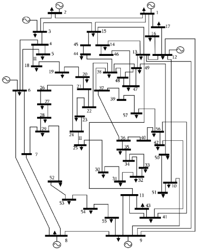


Study Cases

IEEE-57 bus system

- 57 nodes
- 7 generators

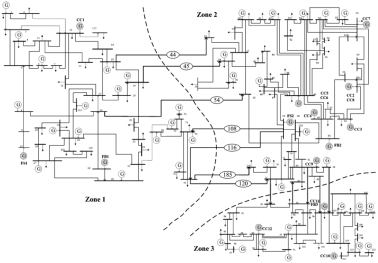
- 80 lines
- 24 time steps



IEEE-118 bus system

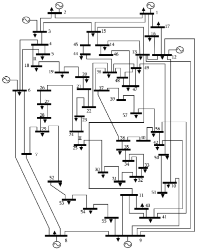
- 118 nodes
- 54 generators

- 186 lines
- 24 time steps



Modified IEEE-57 bus system

- 57 nodes
- 7 generators + 10 generators with VPE
- 80 lines
- 24 time steps



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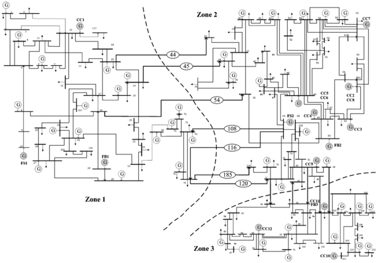

? Where to add the VPE generators **?**

Modified IEEE-57 bus system

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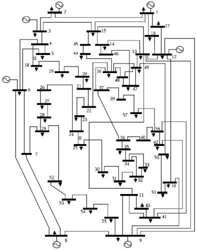

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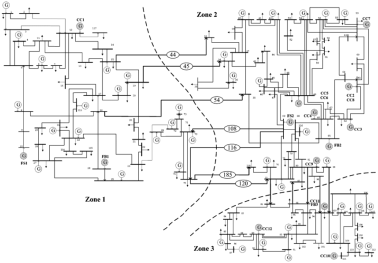
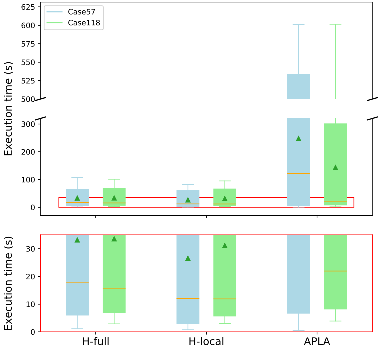


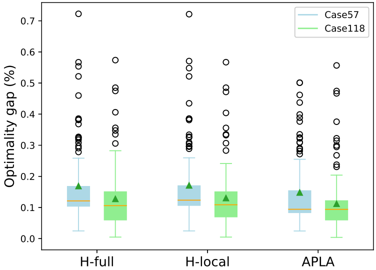
? Where to add the VPE generators **?**

100 randomly independent trials

Execution time and optimality gap

100 independent runs with $n_{\text{iter}} = 1$, $\gamma = 0.1\%$ and time limitation of 10 iterations of maximum 60 seconds





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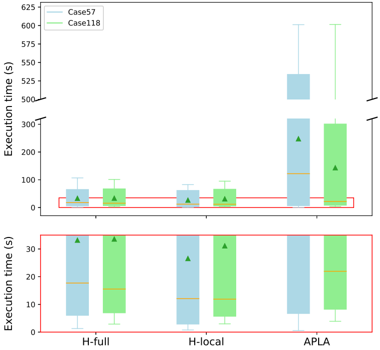

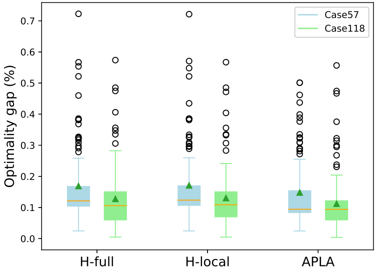

The heuristics run *significantly* faster for comparable finale optimality gap.

Valve Point Effect impact

QP - Solve (P) while *ignoring* the VPE: a simple **Q**uadratic **P**rogramming (convex) problem

Table: Objective mean of each method.

	H-full	H-local	APLA	QP
Case57	621622	621634	621547	654535
Case118	2447475	2447540	2447383	2471132

$$f_g(p_{gt}) = \underbrace{A_g p_{gt}^2 + B_g p_{gt} + C_g}_{:= f_g^{Q}(p_{gt})} + \underbrace{D_g \left|\sin E_g(p_{gt} - P_g^-)\right|}_{:= f_g^{VPE}(p_{gt})}$$

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Best objectives

Valve Point Effect impact

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Nearly as good

⚠ APLA reaches a 10% better optimality gap **but** comparable objective

Valve Point Effect impact

QP - Solve (P) while *ignoring* the VPE: a simple **Q**uadratic **P**rogramming (convex) problem

Table: Objective mean of each method.

	H-full	H-local	APLA	QP
Case57	621622	621634	621547	654535
Case118	2447475	2447540	2447383	2471132

Poorer results (5% and 1%)

Conclusion

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  ... along with a lower bound.
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